

SECTION 7.1 OVERVIEW

A given quantity, amount, or length can be measured in different units. For example, a candy bar that costs \$1.00 also costs 100 cents. A dollar is a larger unit of currency than a cent. Therefore, it takes many cents (100) to equal the value of a single dollar. Similarly, a snake that is 1 foot long is also 12 inches long and it is also 30.48 centimeters long. A foot is a larger unit of length than an inch or a centimeter. It takes more than 30 centimeters to equal the length of a single foot.

It is often necessary to convert numbers that have units in the U.S.C.S. system to numbers with metric units, for example, from pounds to kilograms. It is also often necessary to convert from one U.S.C.S. unit to another U.S.C.S. unit (for example from feet to inches) or from one metric unit to another metric unit (for example from centimeters to meters). This chapter discusses such conversions.

There are two common strategies for performing unit conversions. The first approach is to think of conversion problems in terms of proportions and to use proportion equations to solve them. The second approach has various names including: “unit canceling method” “dimensional analysis” and “factor labeling”. In this text we use the term “unit canceling” method because it aptly describes the strategy. Some people prefer to use proportions to perform conversions, others prefer the unit canceling method. Either strategy will yield the right answers if it is used correctly. As you become experienced with laboratory calculations, you are likely to become comfortable with both strategies.

SECTION 7.2 PROPORTION METHOD OF UNIT CONVERSION

Let's illustrate the proportion approach with an example of a conversion from the U.S.C.S. system to metric units:

If a student weighs 150 pounds, how much does she weigh in kilograms?

First we need a **conversion factor** that relates pounds and kilograms. There are conversion factors in Table 3, Chapter 3 for this purpose. We find in the table that:

$$1 \text{ kilogram} \approx 2.205 \text{ lb}$$

This can be written as a ratio either as:

$$\frac{1 \text{ kilogram}}{2.205 \text{ pounds}} \quad \text{or} \quad \frac{2.205 \text{ pounds}}{1 \text{ kilogram}}$$

Observe that this ratio is equal to 1 since 1 kilogram is the same as 2.205 pounds. All conversion factors are equal to 1.

A proportion equation can now be used to convert from pounds to kilograms:

$$\frac{2.205 \text{ lb}}{1 \text{ kg}} = \frac{150 \text{ lb}}{?}$$

Read this as: "If there are 2.205 pounds in a kilogram, then how many kilograms are 150 pounds"?

Solving for the unknown:

$$? \approx 68.03 \text{ kg} = \text{the student's weight in kg}$$

Examine the answer to see if it makes sense. A kilogram is a larger unit than a pound (just as a dollar is a larger unit than a cent). Therefore, when the student's weight is converted from pounds to kilograms, the number should be lower. Does this make sense to you?

The above problem illustrates how proportion equations can be used to solve unit conversion problems. If we break the process into steps, here is how it is done:

- Step 1. Find a conversion factor that relates the units of interest to one another.**
- Step 2. Express the conversion factor as a ratio (fraction form).**
- Step 3. Express the problem to be solved as a ratio with an unknown.**
- Step 4. Set up a proportion equation such that the units in both numerators are the same and the units in both denominators are the same.**
- Step 5. Solve for the unknown.**
- Step 6. Check that the answer makes sense.**

Here is another example to illustrate these steps:

EXAMPLE PROBLEM:

If a student weighs 80 kg, how much does he weigh in pounds?

ANSWER:

Step 1. Find a conversion factor that relates the units of interest:

$$1 \text{ kg} \approx 2.205 \text{ pounds}$$

Step 2. Express the conversion factor as a ratio (fraction):

$$\frac{1 \text{ kg}}{2.205 \text{ pounds}}$$

Step 3. Express the problem as a ratio with an unknown:

$$\frac{80 \text{ kg}}{?}$$

Step 4. Set up a proportion equation such that the units in both numerators are the same and the units in both denominators are the same:

$$\frac{1 \text{ kg}}{2.205 \text{ pounds}} = \frac{80 \text{ kg}}{?}$$

Read as: "if there are 2.205 pounds per kg, then how many pounds are 80 kg"?

Step 5. Solve for the unknown:

$$\begin{aligned} ? &= \frac{80 \text{ kg} (2.205 \text{ pounds})}{1 \text{ kg}} \\ ? &= 176.4 \text{ pounds} \end{aligned}$$

Step 6. The answer makes sense. A pound weighs less than a kilogram (as a cent is smaller than a dollar) so it takes more pounds than kilograms to equal the student's weight (as it requires more pennies than dollar bills to pay for an item).

EXAMPLE PROBLEM:

A student is 6.00 feet tall. How tall is she in cm? Use these conversion factors:
1 inch = 2.54 centimeters, 1 foot = 12 inches

ANSWER:

This is a proportion problem that, with the conversion factors given, needs to be solved in two parts.

PART 1. Use a proportion expression to convert her height in feet to inches.

Steps 1 and 2 are to find a conversion factor and express it as a ratio:

$$\frac{1 \text{ ft}}{12 \text{ in}}$$

Steps 3 and 4 are to express the problem as a ratio and to set up a proportion expression:

$$\frac{1 \text{ ft}}{12 \text{ in}} = \frac{6.00 \text{ ft}}{?}$$

Read this as "if there are 12 inches in a foot, then how many inches are there in 6 feet"?

Steps 5 and 6:

$$? = 72.0 \text{ in} = \text{her height in inches} \quad \text{This makes sense.}$$

PART 2. Convert the height in inches to height in cm.

Steps 1, 2, 3, and 4:

$$\frac{1 \text{ in}}{2.54 \text{ cm}} = \frac{72.0 \text{ in}}{?}$$

Read this as "if there are 2.54 cm in one inch, then 72 inches is how many cm"?

Steps 5 and 6:

$$? \approx 183 \text{ cm} = \text{her height in cm}$$

A centimeter is a smaller unit than a foot. Therefore, when the student's height is converted from feet to centimeters, the number will have to be larger.

Proportions can also be used to convert from one metric unit to another:

EXAMPLE PROBLEM:

How many meters are 345 cm?

ANSWER:

There are 100 centimeters in a meter. So:

$$\frac{1 \text{ m}}{100 \text{ cm}} = \frac{?}{345 \text{ cm}} \qquad ? = 3.45 \text{ meters}$$

This answer makes sense. A meter is a larger unit than a centimeter. Therefore, when centimeters are converted to meters, the value is a smaller number.

EXAMPLE PROBLEM:

Convert 105 cm to nm.

(1 meter = 10^2 centimeters, and 1 meter = 10^9 nm)

ANSWER:

With the given conversion factors, this problem can be solved using two parts:

$$\text{Part 1.} \quad \frac{1 \text{ m}}{10^2 \text{ cm}} = \frac{?}{105 \text{ cm}} \qquad ? = 1.05 \text{ m}$$

$$\text{Part 2.} \quad \frac{1 \text{ m}}{10^9 \text{ nm}} = \frac{1.05 \text{ m}}{?} \qquad ? = 1.05 \times 10^9 \text{ nm}$$

This answer is reasonable. A nm is a very small unit of length. Therefore, it will take many nanometers to equal the length of 105 cm. The large answer obtained, 1.05×10^9 nm, makes sense.

SECTION 7.3

UNIT CANCELING METHOD OF UNIT CONVERSION

The unit canceling method is a second strategy for doing conversion problems that can be thought of as a shortcut from the proportion method described above. In this method, the number to be converted is multiplied times the proper conversion factor(s). Multiplying the original number times one or more conversion factors does not change the *value* of the number because all conversion factors equal 1. It

does, however, change the *units* of the number, which is the purpose of the procedure. The idea is to pick the conversion factor(s) that cancels out the units you do not want, leaving you with those you do want. (A proportion equation is not used in this strategy). Here is a stepwise overview of this strategy:

Step 1. Write the problem as an equation:

Starting number with starting units = Ending number (*with desired units*)

Step 2. Find one or more conversion factors such that the units cancel, leaving only the desired units.

Step 3. Put the conversion factor(s), expressed as ratio(s), into the equation.

Step 4. Solve the equation.

EXAMPLE PROBLEM:

Convert 2.80 kg to pounds.

ANSWER:

1. Write the problem as an equation:

$$2.80 \text{ kg} = ? \text{ (expressed in pounds)}$$

2. Find a conversion factor such that the units cancel. A pound is 0.454 kg. This conversion factor is expressed as a ratio:

$$\frac{1 \text{ lb}}{0.454 \text{ kg}}$$

(Remember that all conversion factors = 1)

3. Place the conversion factor into the equation:

$$2.80 \text{ kg} \times \frac{1 \text{ lb}}{0.454 \text{ kg}} = ?$$

4. Solve the equation; multiply 2.80 kg by the conversion factor:

$$2.80 \text{ kg} \times \frac{1 \text{ lb}}{0.454 \text{ kg}} \approx 6.17 \text{ lb}$$

Observe that the units of kilograms cancel and the answer comes out with the correct units, pounds.

Suppose you had another reference that told you not that 1 pound is 0.454 kg, but rather that 1 kg is 2.205 pounds. If you use this factor directly, you will get the wrong answer:

$$2.80 \text{ kg} \times \frac{1 \text{ kg}}{2.205 \text{ lb}} \approx 1.27 \text{ kg}^2 / \text{lb}$$

